Markups & Entry in a Circular Hotelling Model

Robert J. Barro
Harvard University
January 2025

Hotelling Model

- Hotelling (1929) constructed model of monopolistic competition with stores located on finite straight line. Consumers purchased goods from nearest store. Because of locational advantages, stores had degree of monopoly power, which entered into pricing decisions.

- Circular version, usually labeled Salop model and attributed to Salop (1979), has major technical advantages because avoids issues related to end points of finite straight line. Present analysis extends by allowing for continuum of consumers with constant-elasticity demand functions along with positive marginal cost of production.

- Focus on model as theory of price markups. Supplements or replaces Lerner approach based on elasticity of demand. Also relates to entry choices.

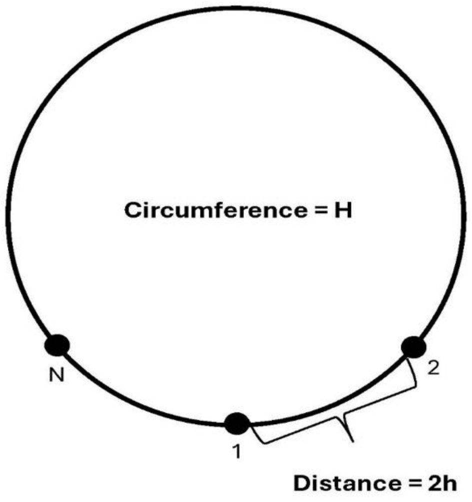
Circular Hotelling Model

- Economy features consumers distributed uniformly around circle with circumference H, as in Figure 1.

- A number N of stores service consumers. Each store j produces goods at same constant marginal cost, c. Store j prices at P_j. In addition to paying P_j for each unit of goods, customer pays amount per unit that increases linearly with distance, z, from store, as in Hotelling. Transportation cost per unit is tz, where $t>0$ is transport-cost parameter that depends on transportation technology and value of time.

- As stressed by Hotelling, "transportation cost" is metaphor for many differentiating characteristics of goods and stores that cause customers to prefer one seller over another for given price charged.

Figure 1
Circular Hotelling Model



Store Spacing

- As in Salop, stores evenly spaced around circle in Figure 1. Even spacing of stores equilibrium outcome in present model. Spacing between stores, denoted $2h$, will satisfy

(1) $$2h = \frac{H}{N}.$$

- Store 1 adjacent to stores 2 and N, as shown. Going to right from its location, store 1 services customers out a distance denoted as h_{12}, which will ultimately equal h, mid-point of distance from store 2.

- Similarly, going to left, store 1 services customers out a distance h_{1N}, which will ultimately equal h. For now, number of stores, N, given. In full equilibrium, N and, hence, $2h$ in Eq. (1) determined from free-entry condition.

Pricing

- Store j chooses price, P_j, for given prices and locations of other stores (as in Bertrand analysis). Without loss of generality, take $j=1$ and consider only market going to right in Figure 1, where store 2 adjacent alternative. (Results that include customers to left, toward store N, analogous.) Assumption is market border between stores 1 and 2, located distance h_{12} from store 1, interior between stores; that is, $0 < h_{12} < 2h$. This assumption later demonstrated to be valid.

- Denote by $q_1(z)$ quantity purchased of store 1's goods by customers located distance z to right of store. Effective price per unit faced by customers at z, where $0 \leq z \leq h_{12}$, adds linear transportation cost, tz, to price set by store:

$$P_1^*(z) = P_1 + tz, \tag{2}$$

- Parameter t same for each store. Assumption is that store 1 charges each customer same price regardless of customer's location.

Lancaster-Baumol

- Transportation costs can be reinterpreted as costs from consuming good with characteristics that deviate from customer's ideal type, as in Lancaster (1966) and Baumol (1967). As noted by Schmalensee (1978 p. 309), "… the formal correspondence between Lancastrian models with two characteristics and one-dimensional spatial models is almost exact."

- Equation (2) implies that transportation charges, tz per unit, proportional to amount transported, $q_1(z)$. In some contexts, such as driving to and from grocery store, charges might be independent of quantity over some range. However, for most applications, reasonable to view transportation charges, including analogous costs in Lancaster-type models, as proportional to quantity.

Constant Elasticity of Demand

- Quantity demanded of store 1's goods by customers located at distance *z* from store assumed to take constant-elasticity form:

(3) $$q_1(z) = A \cdot [P_1^*(z)]^{-\eta},$$

- where $\eta \geq 0$. Standard analysis requires $\eta > 1$, but this restriction ultimately unnecessary in present model. Parameter $A > 0$ represents scale of customer demand. Parameters *A* and η same for each store.

- Hotelling (1929, pp. 45, 56) assumed $\eta = 0$. He says: "The problem … might be varied by supposing that each consumer buys an amount of the commodity in question which depends on the delivered price. If one tries a particular demand function the mathematical complications will now be considerable … " Smithies (1941) and Hay (1976) extended Hotelling to allow for elastic consumer demand, though in form of linear demand functions. Anderson and de Palma (2000) and Gu and Wenzel (2009) allowed for constant elasticity of demand, as in Eq. (3), but their restriction to $\eta < 1$ limits applicability of setting. Also assumed transportation charges independent of quantity transported.

Aggregating Consumer Demand

- Convenient modeling device for aggregating heterogeneous buyers treats $q_1(z)$ as contributing infinitesimal amount to total quantity of goods demanded, Q_1, from store 1:

$$\frac{dQ_1}{dz} = q_1(z).$$

- Total quantity, Q_1, sold by store 1 to customers to right of store's location equals integral of $q_1(z)$ for z going from *0* to market border, h_{12}:

$$Q_1 = \int_0^{h_{12}} q_1(z)dz.$$

- Using demand curve from Eq. (3), integral can be evaluated to get

$$Q_1 = \frac{A}{t(\eta-1)}\left[P_1^{1-\eta} - (P_1 + th_{12})^{1-\eta}\right], \quad (4)$$

where $\eta>1$ assumed at this stage.

Pricing for Given Border Position

- Marginal effect of P_1 on Q_1 follows from Eq.(4) as

(5) $$\frac{\partial Q_1}{\partial P_1} = -\left(\frac{A}{t}\right) \cdot \left[P_1^{1-\eta} - (1 + t\frac{\partial h_{12}}{\partial P_1})(P_1 + th_{12})^{-\eta}\right].$$

- Essence of Hotelling analysis is $\frac{\partial h_{12}}{\partial P_1}$, which indicates how increment in P_1 affects location of market border between stores 1 and 2.

To see relation to standard Lerner formula for markup pricing, useful to begin with counter-factual assumption $\frac{\partial h_{12}}{\partial P_1} = 0$.

- Profit flow for store 1 (for sales to right in Figure 1) is

(6) $$\pi_1 = (P_1 - c)Q_1 - f,$$

where $f > 0$ fixed cost of operating store. Parameter f same for each store. First-order condition for choosing P_1 to maximize π_1 follows from Eq. (6) as

(7) $$\frac{\partial \pi_1}{\partial P_1} = Q_1 + (P_1 - c)\frac{\partial Q_1}{\partial P_1} = 0.$$

Lerner Formula

- Substituting into Eq. (7) for Q_1 from Eq. (4) and $\frac{\partial Q_1}{\partial P_1}$ from Eq. (5) (assuming $\frac{\partial h_{12}}{\partial P_1} = 0$) leads, if $\eta>1$, to first-order maximization condition:

$$(8) \qquad \left(\frac{1}{\eta-1}\right) \cdot \left[P_1^{1-\eta} - (P_1 + th_{12})^{1-\eta}\right] = (P_1 - c)\left[P_1^{-\eta} - (P_1 + th_{12})^{-\eta}\right].$$

- Results simplify if maximal operative transport cost, th_{12}, small compared to P_1; that is, if main component of effective price, $P_1^*(z)$, is always price charged, P_1, rather than transport cost, tz. This approximation, explored in detail in next section, allows for simplifications of terms involving $P_1 + th_{12}$ in Eq. (8) by means of first-order Taylor approximations. These approximations deliver standard result, as in Lerner (1934), whereby P_1 constant markup on marginal cost:

$$(9) \qquad P_1 \approx c \cdot \left(\frac{\eta}{\eta-1}\right).$$

- Second-order condition for profit maximization satisfied if $\eta>1$.

- Atkeson-Burstein (2008) generalize Lerner to have margins of substitution for goods within and across sectors. Overall elasticity of demand is market-share-weighted average of 2 underlying elasticities and need not be constant. Magnitude of overall elasticity exceeds 1 because magnitude of 2 underlying elasticities each assumed to exceed 1.

Hotelling Effect at Borders

- At border between markets for stores 1 and 2, where $z=h_{12}$, customers indifferent between buying goods from store 1 or 2. For this indifference to apply, prices inclusive of transportation costs, P_1^* and P_2^*, must be equal at border:

(11) $$P_1 + th_{12} = P_2 + t(2h - h_{12}),$$

where $2h - h_{12}$, assumed positive, is distance of market border from store 2. Starting from position where Eq. (11) holds, increment in P_1, given P_2, requires compensating adjustment of border position, h_{12}:

(12) $$\frac{\partial h_{12}}{\partial P_1} = -\frac{1}{2t}.$$

Markup Pricing

- Substituting result from Eq. (12) into expression for $\frac{\partial Q_1}{\partial P_1}$ in Eq. (5) implies

(13) $$\frac{\partial Q_1}{\partial P_1} = -\left(\frac{A}{t}\right) \cdot [P_1^{-\eta} - \frac{1}{2}(P_1 + th_{12})^{-\eta}].$$

- Substitution into expression for $\frac{\partial \pi_1}{\partial P_1}$ in Eq. (7) yields new first-order maximization condition:

(14) $$\left(\frac{1}{\eta-1}\right) \cdot \left[P_1^{1-\eta} - (P_1 + th_{12})^{1-\eta}\right] = (P_1 - c) \cdot [P_1^{-\eta} - \frac{1}{2}(P_1 + th_{12})^{-\eta}].$$

- Results again simplify if maximal operative transport cost small compared to price; that is, $th_{12} << P_1$. Then Eq. (14) simplifies using first-order Taylor approximations for terms involving $P_1 + th_{12}$ to

(15) $$P_1 \approx c + 2th_{12}.$$

More on Markup Pricing

- Second-order condition for profit maximization satisfied if $\eta \geq 0$.

- Solution in Eq. (15) holds even when $\eta \leq 1$. In standard, Lerner analysis, this range for η excluded to avoid infinite prices and profits by stores. Contraction of market border when price rises allows for inclusion of $\eta \leq 1$.

- Can relate market border position, h_{12}, to spacing between stores, given by $2h$ in Figure 1. Eq. (15) holds for pricing by store 1 and analogous condition holds for pricing by store 2:

(16) $$P_2 \approx c + 2t \cdot (2h - h_{12}).$$

- Combining Eqs. (15) and (16) with Eq. (11) yields intuitive result that border equidistant between stores 1 and 2, $h_{12} = h$. Correspondingly, prices charged by two stores the same:

(17) $$P_1 = P_2 = P \approx c + 2th.$$

Price Undercutting

- Results assumed distance from store 1 to market border, h_{12}, interior between stores 1 and 2, so that $0 < h_{12} < 2h$. Another possibility is that store 1 undercuts store 2 by pricing so as to move border, h_{12}, up to or beyond $2h$.

- To be attractive to buyers at store 2's location (and at locations further beyond store 2), P_1 has to be set below P_2 by at least extra transport cost, $2th$. But if $P_2 = c + 2th$, as in Eq. (17), this undercutting means P_1 must be set at or below marginal cost, c. Outcome unprofitable for store 1. Therefore, satisfactory to assume interior equilibrium where $0 < h_{12} < 2h$. (Vogel [2008] has broader analysis of this type of undercutting in model of spatial competition.)

Approximations in Pricing Formula

- Table A1 in appendix examines accuracy of approximations in Eq. (17). Eq. (14) solved numerically to get solution for P_1—that is, P— given specification of underlying parameters, which can be expressed as η and $2th/c$. Latter is ratio of markup to marginal cost in formula in Eq. (17). Note from Eq. (14) that Eq. (17) holds exactly if $\eta=0$. Also exact if $th=0$.

- More generally, as shown in Table A1, part 1, equation more closely approximates true solution for P when η and $2th/c$ smaller. For ranges of parameters that seem "reasonable"— such as $\eta \leq 5$ and $2th/c \leq 0.1$ or $\eta \leq 2$ and $2th/c \leq 0.2$—solution for P falls short of $c+2th$ by less than 3%. Therefore, in plausible ranges for parameters, formula for P in Eq. (17) yields close approximation to true solution.

- Table A1, part II compares numerical solution for P with that from Lerner formula, $P = c \cdot \left(\frac{\eta}{\eta-1}\right)$, well defined only if $\eta>1$. Lerner formula provides good approximation only for very large η. Formula also fits better the larger $2th/c$. However, in ranges of "reasonable" parameter values noted before, Lerner formula gives poor results.

Pass-Through in Levels

- Another way to look at pricing result in Eq. (17) is, for given *2th*, change in marginal cost, c, passes through one-to-one to P. In language of Sangani (2024), model predicts 100% pass-through in levels.

- Empirical analysis in his paper (Tables 4 and 5) confirms this relationship for retail gasoline and array of retail food prices. (Results for excise taxes?) In contrast, Lerner formula implies P proportional to c for given η. Sangani refers to this case as 100% pass-through in logs and shows it is inconsistent with data.

Lerner Formula in Markup Literature

- Many studies in literature on price markups mention Lerner formula but make no use of it empirically. Examples include Hall (2018), Bond, et al. (2021), Syverson (2024). Given poor performance of Lerner formula in Hotelling-type models (Table A1, part II), makes sense not to rely on this formula for empirical analyses of price markups.

- Instead, recent applied literature, exemplified by de Loecker, Eeckhout, Unger (2020), focuses on how to measure price markups with available data, following approach pioneered by Hall (1988). Although results useful, do not relate markups to "fundamentals," which include elasticity of demand and factors that affect substitutability of neighboring products. In Hotelling-type model, latter group comprises influences on spacing between stores (or products), $2h$, and transportation-cost parameter, t.

Full Elasticity of Demand

- Another way to view results is in terms of model's prediction about size of full elasticity of Q with respect to P. When formula for P in Eq. (17) satisfactory, Q from Eq. (4) can be approximated as

- (18) $$Q \approx AhP^{-\eta}.$$

- Full elasticity of Q with respect to P involves usual -η term, along with additional negative effect from decrease in market border, h, based on Eq. (12) (along with h_{12}=h). Result for full elasticity is

- (19) $$\frac{\partial Q}{\partial P} \cdot \frac{P}{Q} \approx -\eta - \frac{c+2th}{2th}.$$

More on Elasticity of Demand

- Standard Lerner analysis, which ignores reduction in h, requires $\eta>1$ to get meaningful results. However, since magnitude of final term on right side of Eq. (19) exceeds 1 (by a lot if $c>>2th$), magnitude of full elasticity exceeds 1 for any $\eta \geq 0$. Hence, market-border effect in Hotelling model avoids having to assume $\eta>1$, which need not hold. Parallel to overall elasticity of demand in Atkeson-Burstein? Identify cross-sector elasticity with my η and within-sector elasticity with my market-border effect? But A-B assume magnitude of both underlying elasticities exceeds 1.

- Price elasticity in Eq. (19) does not apply to overall quantity of goods. Quantity Q in Eq. (18) is amount sold by individual store on one side of location. Quantity sold on both sides multiplied by 2, and overall quantity in whole market multiplied by number of stores, N.

- Using Eqs. (18) and (1), overall quantity approximately $AHP^{-\eta}$. Hence, with H given, η is price elasticity for overall quantity. For example, for groceries, η corresponds to price elasticity of demand for groceries in aggregate (likely small), whereas object in Eq. (19) is full price elasticity of demand for individual grocer, holding fixed prices and locations of other grocers.

Equal Spacing of Stores

- Return to assumption that stores evenly spaced around circle in Figure 1. Suppose that store 1 takes as given positions of stores 2 and *N*. Suppose, starting from position equidistant between stores *N* and 2, that store 1 considers moving its location, say by distance *X* to right. Market border with store *N* shifts from distance *h* to *h+X/2* from store 1's location. Border with store 2 shifts from distance *h* to *h-X/2*. Therefore, total distance covered by store 1 remains at *2h*.
- However, shift toward customers relatively far from store (in region toward store *N*) and away from those relatively close (in region toward store 2). Because of downward-sloping demand curve, more distant customers buy smaller quantity and are less profitable. Hence, on net, store 1's profit declines. It follows that store 1 would not move and is best off remaining equidistant between stores 2 and *N*. That is, equal-spacing pattern is equilibrium outcome.
- Note that this conclusion depends on downward-sloping demand curve—result would not apply under common assumption that each household buys exactly one unit of good. Result also depends on assumption that transportation charges proportional to quantity bought, *q(z)*, rather than being independent of quantity.
- Result accords with Vickrey (1964).

Free-Entry Condition

- This analysis of entry applies when $P \approx c+2th$. Total quantity sold by each store is

(18) $$Q \approx 2hAc^{-\eta}.$$

Associated profit is

(19) $$\pi \approx 4tAh^2c^{-\eta} - f.$$

If $\pi > 0$, incentive for new stores to enter, thereby raising N and lowering $h = \frac{1}{2}\frac{H}{N}$. If integer constraint on N can be neglected, as will be satisfactory if N large, free-entry condition will be $\pi \approx 0$. Eq. (19) then implies

(20) $$2h \approx \sqrt{\frac{f}{tAc^{-\eta}}}.$$

Properties of Free-Entry Result

- Under free entry, spacing between stores, $2h$, follows square-root rule; spacing larger the lower transport cost per unit, t, the higher fixed cost f of operating a store, and the lower (approximate) quantity sold to each buyer, $Ac^{-\eta}$. (Fixed cost is effectively scaled by quantity sold to each buyer.) Spacing does not depend on H.

- Number of stores is

$$N = \frac{H}{2h} \approx H \cdot \sqrt{\frac{tAc^{-\eta}}{f}}. \tag{21}$$

- Price charged is

$$P \approx c + \sqrt{\frac{tf}{Ac^{-\eta}}}. \tag{22}$$

P rises roughly one-to-one with c and still depends to a minor extent on η.

Empirical Findings of Chevalier, et al.

- Theoretical results relate to empirical analysis of Chevalier, Kashyap, Rossi (2003) concerning pricing patterns of major supermarket chain. Key finding is markups for affected categories of products relatively low at times of peak demand, notably during major holidays and events such as Christmas, Thanksgiving, Lent. Empirical results rule out (Lerner-type) explanation for pricing pattern based on elasticity of demand being unusually high at times of peak demand. Findings accord with present model in that price markup not predicted to fall when demand elasticity, η, rises for given market size, $2h$, or when $2h$ determined by free-entry condition.

- In model, peak demand represented by a temporarily high parameter A, which enters into demand function. High A has no effect on price markup in Eq. (17) for given market size, $2h$, but reduces markup in Eq. (22), which factors in free-entry condition. An application of last result to Chevalier, Kashyap, Rossi depends on significant entry during peak demand periods. Possibly this entry can take form not of new stores but of longer store hours or increased advertising (margin emphasized by Chevalier, Kashyap, Rossi).

Socially-Optimal Entry

- Heuristically, two elements in model may cause free-entry choices of spacing, *2h*, and number of stores, *N*, to deviate from socially-optimal values. First distortion is markup, approximated by *2th*, which generates excess of effective price, P^*, over social marginal cost, $c+tz$. Excessive price leads to quantities of goods consumed falling short of socially-optimal values. Therefore, profit flow, signal for entry, too low in sense of falling short of amount that corresponds to socially-optimal quantity of goods produced. On this ground, one would expect free-entry choice of number of stores, *N*, to be too low, corresponding to spacing, *2h*, being too high.

- Second distortion is that entering firm's profit includes revenue transferred from incumbent firms. This is "business-stealing effect," which is private reward for entry that has no social benefit. (Distortion can be viewed as generated by lack of property rights for incumbent firms in profit flows.) On this ground, one would expect free-entry choice of number of stores, *N*, to be too high, corresponding to spacing, *2h*, too low.

Excess Entry

- Tirole's (1988) analysis of Salop-Hotelling model finds entry excessive in that model. He allows only for business-stealing effect because $\eta=0$ assumed.

- Present model also implies excess entry when approximation $P \approx c+2th$ satisfactory. Allows for $\eta \neq 0$ but η small enough that first form of distortion comparatively minor. Entry need not be excessive when η (or $2th$) very large.

Conclusion on Price Markups

- From perspective of understanding price markups, biggest contribution from Hotelling-type models is emphasis on factors that influence substitutability among neighboring producers and products. This focus contrasts with standard Lerner formula, which puts all weight on elasticity of demand.