The ultralong sovereign default risk

Radek Paluszynski

Department of Economics
University of Houston

August 2024

Introduction

- Long tradition of thinking about sovereign debt maturity in economics.
- Positive analyses and normative prescriptions.

Introduction

- Long tradition of thinking about sovereign debt maturity in economics.
- Positive analyses and normative prescriptions.
- **Ultralong bonds:** recent emergence of old-new instrument.
- Borrowers range from Mexico to Japan, nontrivial amounts.
- Why issue such securities, and why now?

Introduction

- Long tradition of thinking about sovereign debt maturity in economics.
- Positive analyses and normative prescriptions.
- **Ultralong bonds:** recent emergence of old-new instrument.
- Borrowers range from Mexico to Japan, nontrivial amounts.
- Why issue such securities, and why now?
- Special role of default risk:

  Foreign creditors are more bullish (...) Those are extraordinarily good terms given Mexico's distinctly spotty credit record.

The Economist, 2015

This paper

1. Evidence
   - Issuance of ultralong bonds over time and interest rates
2. Sovereign default model with endogenous maturity and variable risk-free rate
   - Ultralong bonds useful to hedge against low-frequency movements in interest rates.
   - Benefit of this insurance largest when rates are **persistent** and **low**.
   - Model calibrated to Mexico's default history captures the recent ultralong issuances and corresponding spreads.
   - Default risk helps motivate the hedging particularly when rates are low.

Mechanism

Hedging benefit of long-term debt vs. incentive benefit of short-term debt

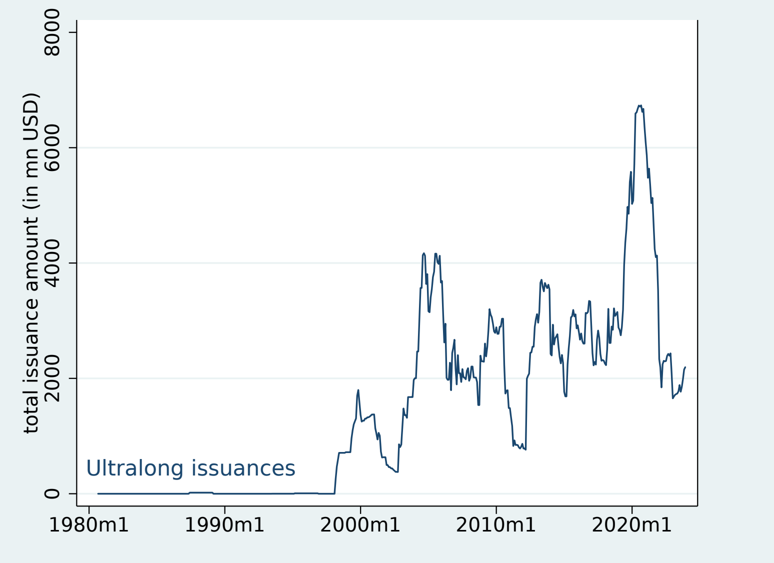
- Positive hedging benefit of ultralong debt:
  - insulates borrowers from shocks to risk-free rate
  - especially beneficial if such shocks are very persistent...
- Negative incentives-to-default properties of ultralong debt:
  - borrower less willing to reduce debt in the future to attain better bond prices
  - increasing future default risk

Other forces (not here)

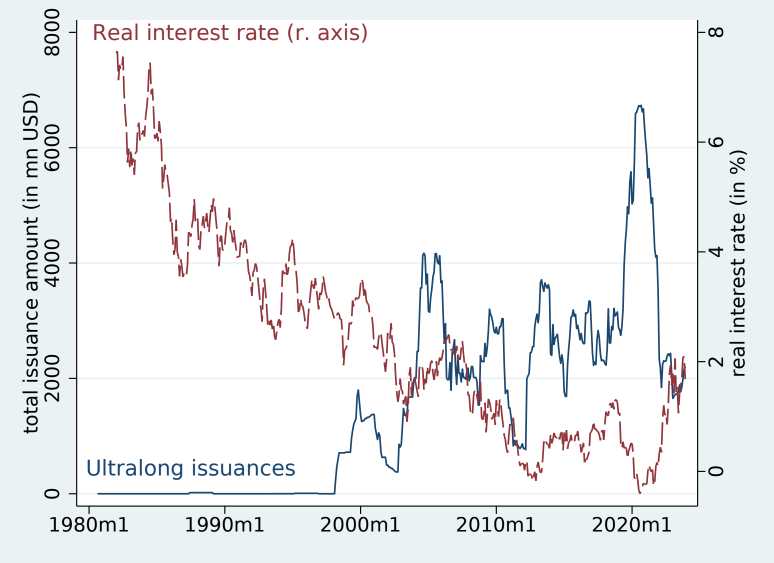
- Demand structure and clientele effects
  - Buyer of ultralong bonds: pension funds, life insurance companies, etc.
- Default by inflation and sovereign market power
- Lack of commitment in fiscal policy (Lucas and Stokey, 1983)

Evidence

Ultralong issuances

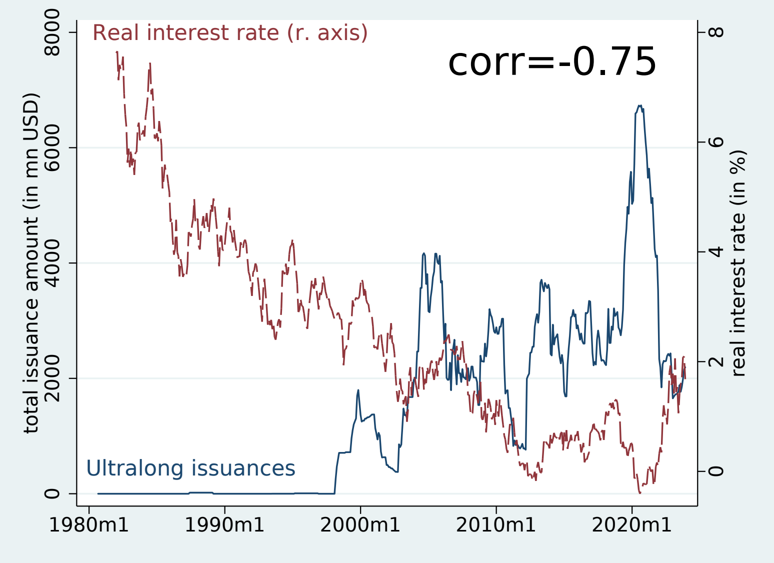


Ultralong issuances and risk-free rate



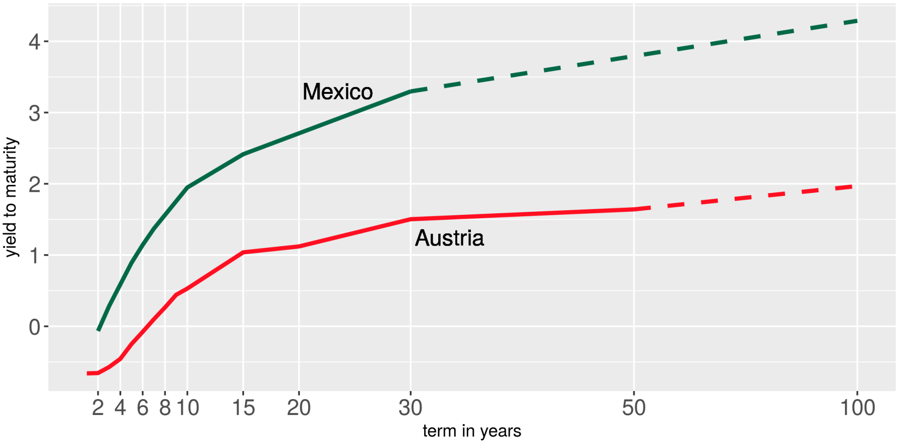
Countries | Maturity × Country | Maturity × Iss. amt | Iss. and tm prm | MX: share of debt | Own currency issuers

Ultralong issuances and risk-free rate



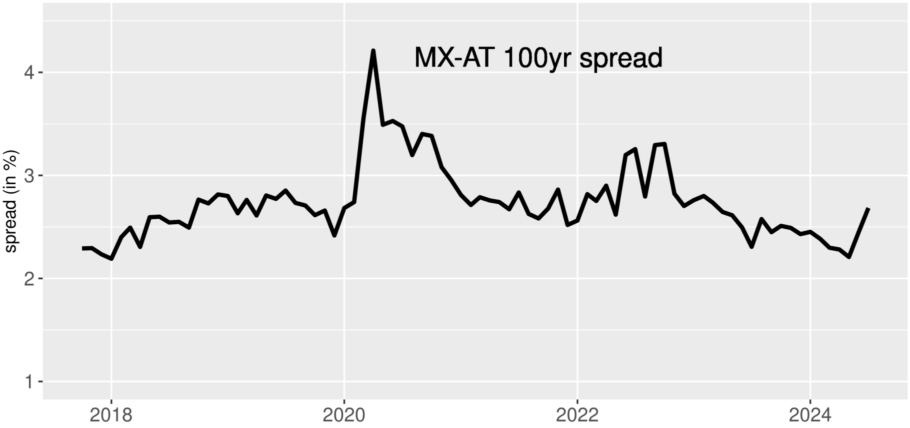
Countries | Maturity × Country | Maturity × Iss. amt | Iss. and tm prm | MX: share of debt | Own currency issuers

Mexico-Austria yield curves



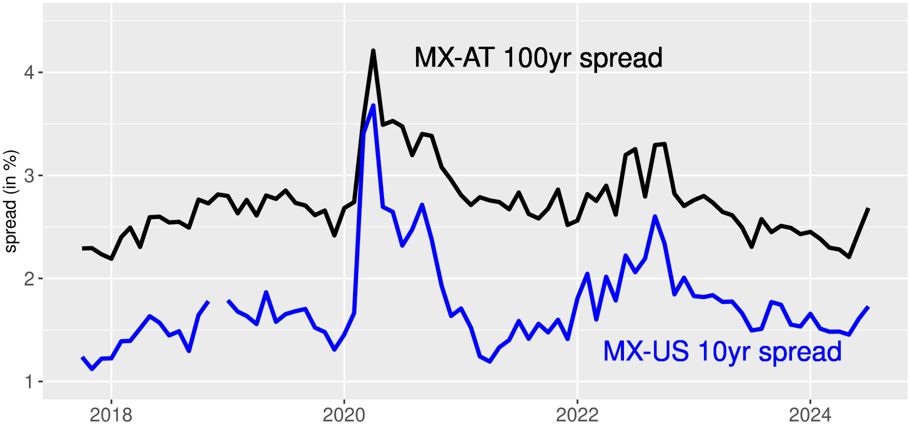
Source: Bloomberg.

Mexico-Austria 100 year spread



Source: Bloomberg. MX-US yields

Mexico-Austria 100 year spread



Source: Bloomberg. MX-US yields

Model

Model overview

- Small open economy with stochastic processes for income **and risk-free rate**.
- Gov't acts on behalf of households to smooth out and front-load consumption.
- **Two types** of defaultable debt: shorter and longer maturity
- Two punishments in default:
  - Direct cost to income
  - Temporary exclusion from credit markets
- Zero recovery rate for debt following a settlement of default
- International lenders are perfectly competitive and risk neutral.

Stochastic processes

- Time is discrete, $t = 0, 1, 2, \ldots$, each period equal to one year.
- Economy receives a stochastic endowment Y_t such that

$$log\ Y_t = \sum_{s=1}^{t} g_s$$

- Income growth rate is i.i.d. around a mean.

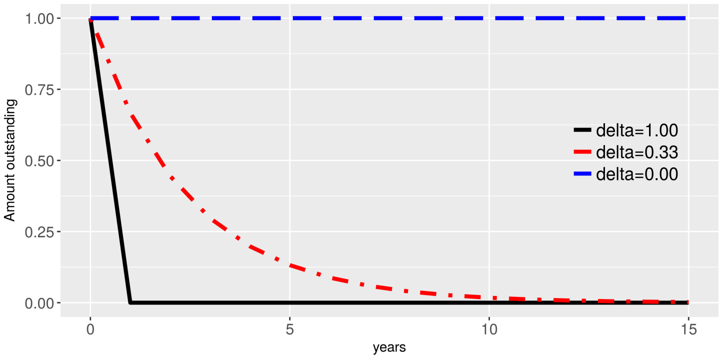
$$g_t = \mu + \sigma \varepsilon_t$$

- Risk-free interest rate follows an AR(1) process.

$$r_{t+1} = (1 - \rho_r)\bar{r} + \rho_r r_t + \sigma_r \varepsilon_{r,t+1}$$

Market structure

- Bonds are perpetuities with declining coupon payments over time.
- Each bond collapsed to state variable B_i with decay parameter δ_i.



Sovereign's problem

Aggregate states: B, B_u, S, where $\mathsf{S} = (Y_{-1}, g, r)$.

Sovereign's problem

Aggregate states: B, B_u, S, where $\mathsf{S} = (Y_{-1}, g, r)$.

Choice to repay or default:

$$v(B, B_u, \mathsf{S}) = \max_{d \in \{0,1\}} \left\{ (1-d) v^r(B, B_u, \mathsf{S}) + d v^d(B, B_u, \mathsf{S}) \right\}$$

Sovereign's problem

Aggregate states: B, B_u, S, where $\mathsf{S} = (Y_{-1}, g, r)$.

Choice to repay or default:

$$v(B, B_u, \mathsf{S}) = \max_{d \in \{0,1\}} \Big\{ (1-d) v^r(B, B_u, \mathsf{S}) + d v^d(B, B_u, \mathsf{S}) \Big\}$$

Value of repayment:

$$v^r(B, B_u, \mathsf{S}) = \max_{C, B', B'_u} \Big\{ u(C) + \beta\, \mathbb{E}\Big[v(B', B'_u, \mathsf{S}') \,|\, \mathsf{S} \Big] \Big\}$$

subject to

$$C = Y - B - B_u\Big(\delta_u + (1-\delta_u)\kappa\Big) + q(B', B'_u, \mathsf{S})\Big(B' - (1-\delta)B\Big) + q_u(B', B'_u, \mathsf{S})\Big(B'_u - (1-\delta_u)B_u\Big)$$

Sovereign's problem

Aggregate states: B, B_u, S, where $\mathsf{S} = (Y_{-1}, g, r)$.

Choice to repay or default:

$$v(B, B_u, \mathsf{S}) = \max_{d \in \{0,1\}} \Big\{ (1-d) v^r(B, B_u, \mathsf{S}) + d v^d(B, B_u, \mathsf{S}) \Big\}$$

Value of repayment:

$$v^r(B, B_u, \mathsf{S}) = \max_{C, B', B'_u} \Big\{ u(C) + \beta \, \mathbb{E}\Big[v(B', B'_u, \mathsf{S}') \,|\, \mathsf{S} \Big] \Big\}$$

subject to

$$\begin{aligned} C = Y - B - B_u\Big(\delta_u + (1-\delta_u)\kappa\Big) \\ + q(B', B'_u, \mathsf{S})\Big(B' - (1-\delta)B\Big) + q_u(B', B'_u, \mathsf{S})\Big(B'_u - (1-\delta_u)B_u\Big) \end{aligned}$$

Value of default:

$$v^d(\mathsf{S}) = u\Big(Y(1-\phi)\Big) + \beta \, \mathbb{E}\Big[\theta \, v(0, 0, \mathsf{S}') + (1-\theta) \, v^d(\mathsf{S}') \,|\, \mathsf{S}\Big]$$

Bond prices

For regular maturity bonds:

$$q(B', B'_u, \mathsf{S}) = \frac{1}{1+r}\mathbb{E}\Big[\ \Big(1 - d(B', B'_u, \mathsf{S}')\Big)\Big[1 + (1-\delta)q(B'', B''_u, \mathsf{S}')\Big] \,|\, \mathsf{S}\Big]$$

For ultralong bonds:

$$q_u(B', B'_u, \mathsf{S}) = \frac{1}{1+r}\mathbb{E}\Big[\ \Big(1 - d(B', B'_u, \mathsf{S}')\Big)\Big[\delta_u + (1-\delta_u)\Big(\kappa + q_u(B'', B''_u, \mathsf{S}')\Big)\Big] \,|\, \mathsf{S}\Big]$$

Intuition: why issue long-term debt?

Stylized Euler equation for a choice of debt level with decay rate δ_i :

$$
\begin{aligned}
&u'(C)\Big[q_i + \frac{\partial q_i}{\partial B_i'}\big(B_i' - (1-\delta_i)B_i\big) + \frac{\partial q_j}{\partial B_i'}\big(B_j' - (1-\delta_j)B_j\big)\Big] \\
&= \mathbb{E}\Big\{u'(C')\Big[1 + (1-\delta_i)q_i'\Big]\Big\} \\
&= \mathbb{E}u'(C')\,\mathbb{E}\Big[1 + (1-\delta_i)q_i'\Big] + (1-\delta_i)cov\Big(u'(C'),\, q_i'\Big)
\end{aligned}
$$

Trade-off between incentive benefit of shorter debt and hedging benefit of longer-term debt (Arellano and Ramanarayanan, 2012).

Intuition: why issue long-term debt?

Stylized Euler equation for a choice of debt level with decay rate δ_i :

$$
\begin{aligned}
&u'(C)\Big[q_i + \frac{\partial q_i}{\partial B_i'}\big(B_i' - (1-\delta_i)B_i\big) + \frac{\partial q_j}{\partial B_i'}\big(B_j' - (1-\delta_j)B_j\big)\Big] \\
&= \mathbb{E}\Big\{u'(C')\Big[1 + (1-\delta_i)q_i'\Big]\Big\} \\
&= \mathbb{E}u'(C')\,\mathbb{E}\Big[1 + (1-\delta_i)q_i'\Big] + (1-\delta_i)cov\Big(u'(C'),\, q_i'\Big)
\end{aligned}
$$

Trade-off between incentive benefit of shorter debt and hedging benefit of longer-term debt (Arellano and Ramanarayanan, 2012).

Ultralong debt can hedge against low-frequency movements in RF rate, but also reduces incentives to repay faster.

Quantitative analysis

Stochastic processes

Growth equation for 1980-2023: $g_t = \mu + \sigma \varepsilon_t$

Parameter	μ	σ
Value	0.021	0.037

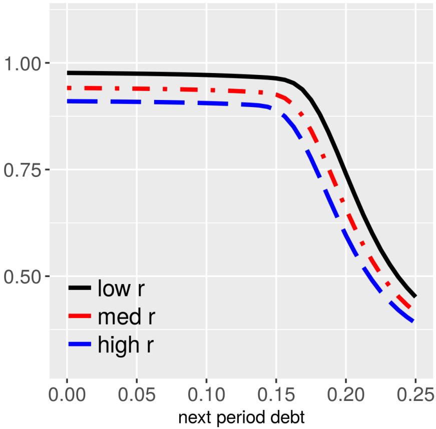
Risk-free rate equation for 1955-2023: $r_{t+1} = (1 - \rho_r)\bar{r} + \rho_r r_t + \sigma_r \varepsilon_{r,t+1}$

Parameter	$\bar{r}$	ρ_r	σ_r
Value	0.022	0.860	0.009

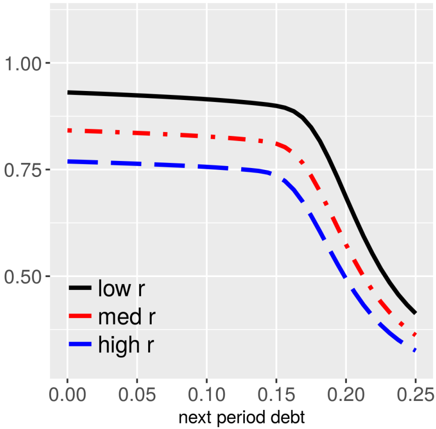
Structural parameters

Symbol	Meaning	Value	Source
γ	Risk aversion	2	Standard
θ	Re-entry probability	0.33	Standard
δ	Decay rate – regular	0.285	} Mexican
δ_u	Decay rate – ultralong	0.03	} debt data
κ	Ultralong coupon rate	0.14	Normalization
ϕ	Default cost	0.07	} Joint
β	Discount factor	0.76	} calibration
Calibration targets		**Model**	**Data**
E (debt/GDP)		21.16	22.00
Default probability		3.00	3.00

Equilibrium bond prices

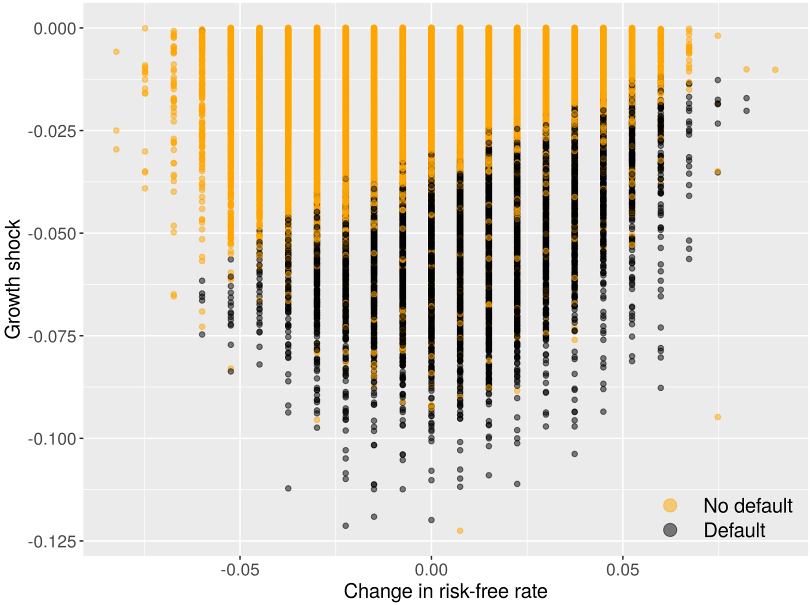


(a) Regular bonds



(b) Ultralong bonds

What generates defaults?



Maturity choices and bond spreads

Table: Untargeted moments

	Mean
ultra share	5.8
spread reg	3.03
spread ultra	2.91
spread term premium	-0.29

Maturity choices and bond spreads

Table: Untargeted moments

	Mean	
ultra share	5.8	
spread reg	3.03	
spread ultra	2.91	
spread term premium	−0.29	
Correlation with:	r	g
ultra share	−0.53	0.003
new ultra share	−0.28	−0.03
spread reg	0.24	−0.31
spread ultra	0.79	−0.15
spread term premium	0.25	0.32

Determinants of hedging

	Mean	St. dev.
q	2.97	0.13
q_u	2.09	0.18
$\left[1+(1-\delta)q\right]b$	0.33	0.03
$\left[\kappa+(1-\delta_u)q_u\right]b_u$	0.01	0.01
Interest rate:	**below mean**	**above mean**
$\text{Corr}\big(u'(c'), q'\big)$	-0.40	-0.40
$\text{Corr}\big(u'(c'), q'_u\big)$	-0.40	-0.34
E (ultra share)	6.3	4.4
E (new ultra share)	2.0	0.6

- Hedging benefit is largest when risk-free rate is below mean

Alternative calibrations

	baseline	high d/low s	constant r	i.i.d. r
debt/GDP	21.2	35.3	20.7	21.7
default probability	3.0	1.5	3.0	2.9
ultra share	5.8	5.2	0.0	0.0
corr(share,r)	−0.53	−0.15	–	–
corr(new share,r)	−0.28	−0.04	–	–

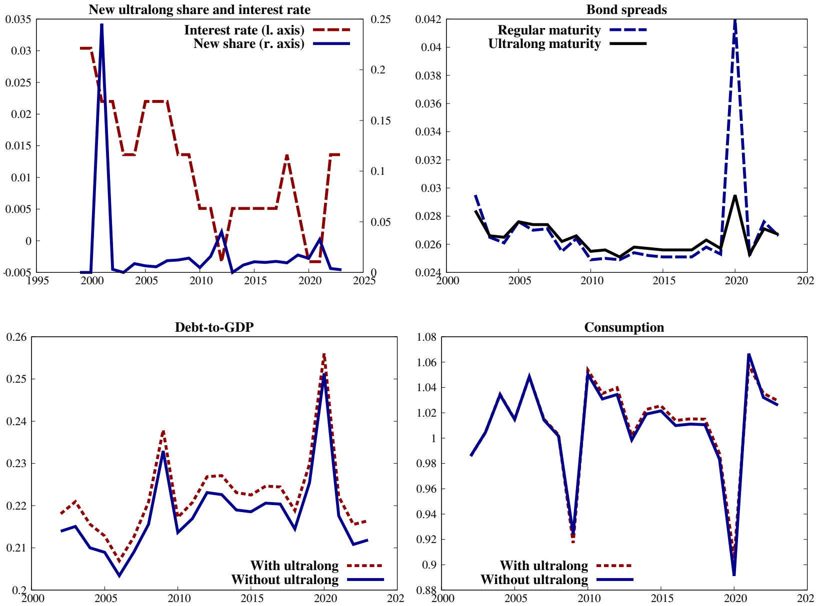
- Default risk important in driving the correlation btw r and ultra share
- High persistence in r needed to generate realistic ultra shares

Gains from ultralong bonds

Metric	With ultra bonds	Without
Frequency of Defaults	2.97%	3.33%
$\mathbb{E}(debt/income)$	21.16%	20.90%
$SD\ log(cons)$ / $SD\ log(y)$	1.20	1.21
$SD\ (TB/y)$	0.23	0.24
$corr(spread, r)$	0.24	-0.11
Average consumption loss		-0.11%
Cons-eqv. welfare loss		-0.37%

- More defaults, higher volatility, less insurance in the world without ultralong bonds

Mexico exercise
Extended model



Conclusion

- Ultralong sovereign debt: old-new instrument in financial markets
- Model of sovereign government choosing to issue such bonds
- Hedging against low-frequency movements in interest rates motivates the issuance of ultralong bonds
- Model can rationalize the recent issuances for Mexico
- Default risk actually increases the hedging motive when rates are low

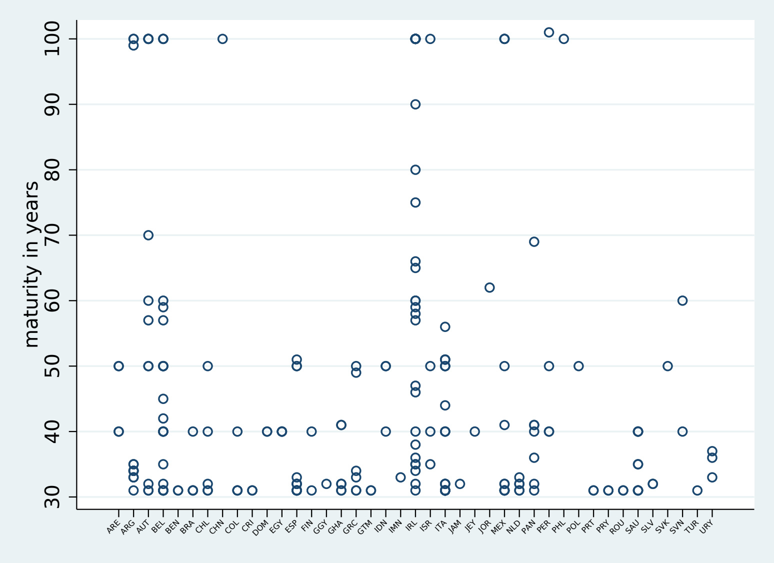
Appendix: additional graphs

Ultralong debt issuers (as of 12/2023)

Main graph

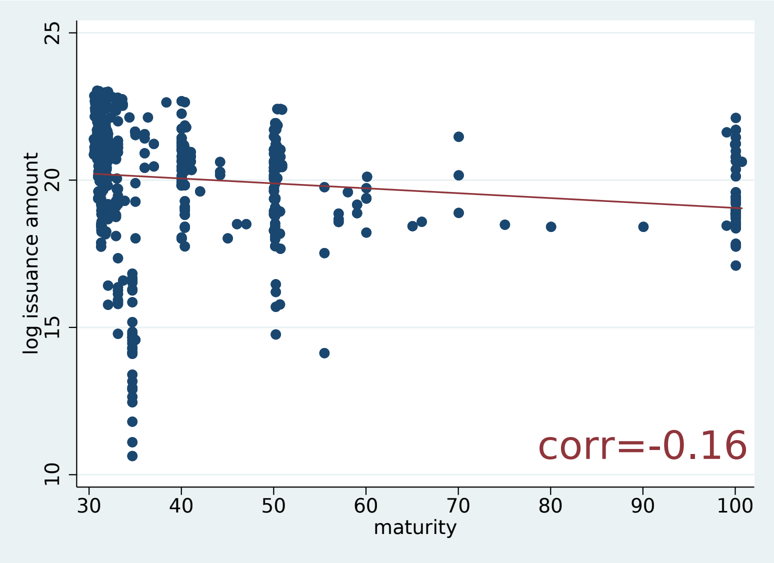
Country	Ave maturity	Amount	% ext. debt
Argentina	93.50	0	0
Austria	71.16	24.08	9.76
Belgium	41.91	25.93	7.30
Chile	40.64	3.00	6.35
China	72.08	0.10	0.02
Colombia	37.17	1.30	1.67
Costa Rica	30.92	3.00	24.82
Dominican Rep.	36.08	6.40	*N/A*
Egypt	36.85	4.00	4.71
Ghana	37.25	1.50	*N/A*
Greece	33.58	1.09	0.36
Indonesia	44.36	2.45	1.25
Ireland	74.78	3.35	2.40
Israel	45.06	8.09	13.49
Italy	44.09	20.76	2.34
Mexico	64.91	11.71	5.35
Panama	35.59	6.95	*N/A*
Peru	54.92	4.00	10.34
Philippines	73.50	0.10	0.14
Poland	31.58	0.55	0.47
Saudi Arabia	34.76	16.00	16.97
Slovakia	44.50	0.74	1.99
Slovenia	47.71	1.09	4.34
Spain	44.60	20.25	2.85
UAE	41.65	7.00	*N/A*
Uruguay	31.33	2.59	11.79

Maturity by country



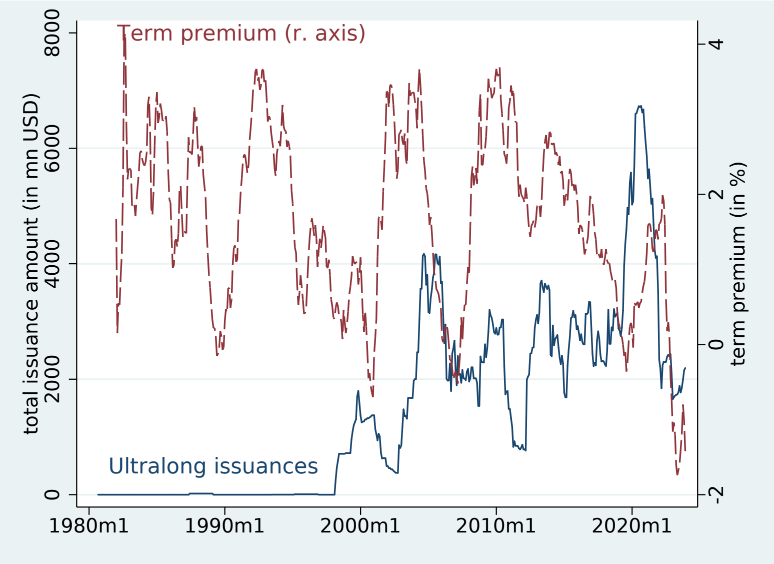
Back to main graph

Maturity by issuance amount



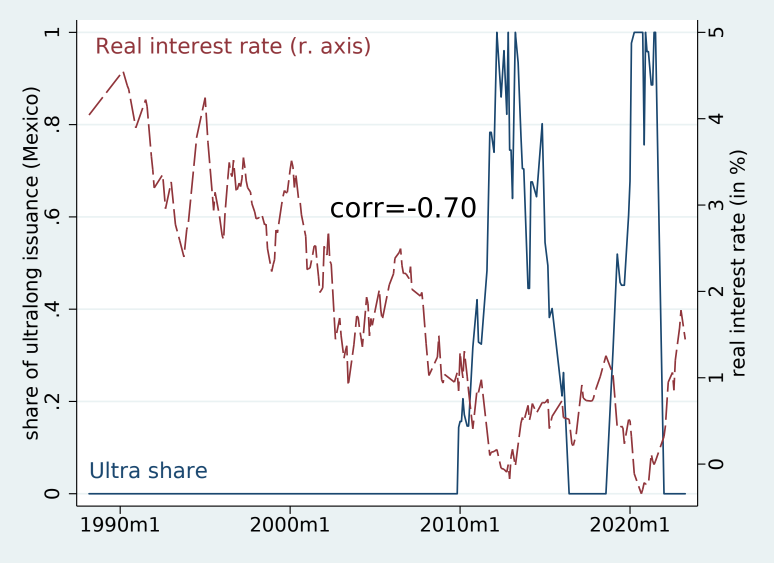
Back to main graph

Ultralong issuances and term premium



Back to main graph

Mexico: new issuance share and risk-free rate



Back to main graph

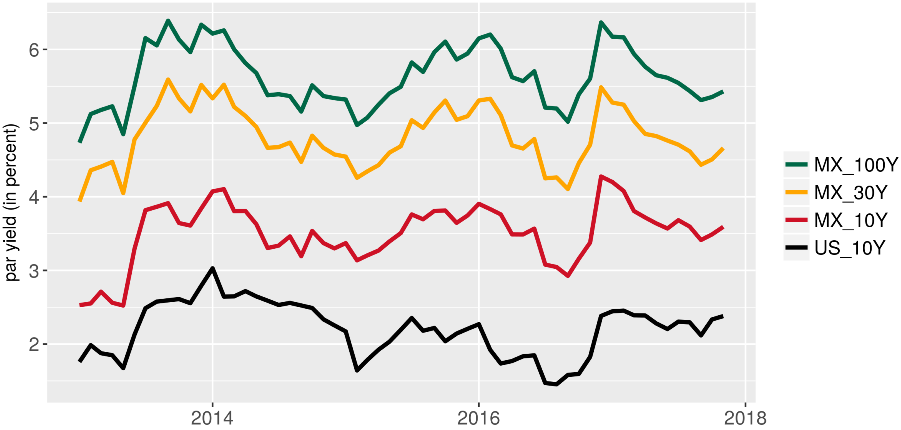
Own currency ultralong debt issuers

Country	Ave maturity > 30	% debt
UK	38.2	11.6
Switzerland	35.4	9.6
Japan	35.5	2.7
Euro Area (DE+FR)	37.0	2.0
US	0	0

Source: Bloomberg. Average maturities are weighted by face values.

Back to main graph

Mexico yields over time



Source: Bloomberg.

MX-AT yield curves

Spreads under risk-averse lenders

Back

Risk-averse lenders

- Two-factor affine term structure model.
- $M(s, s') = exp\{-r_t - \frac{1}{2}a_t^2\sigma_x^2 - a_t\sigma_x\varepsilon_{x,t+1}\}$.

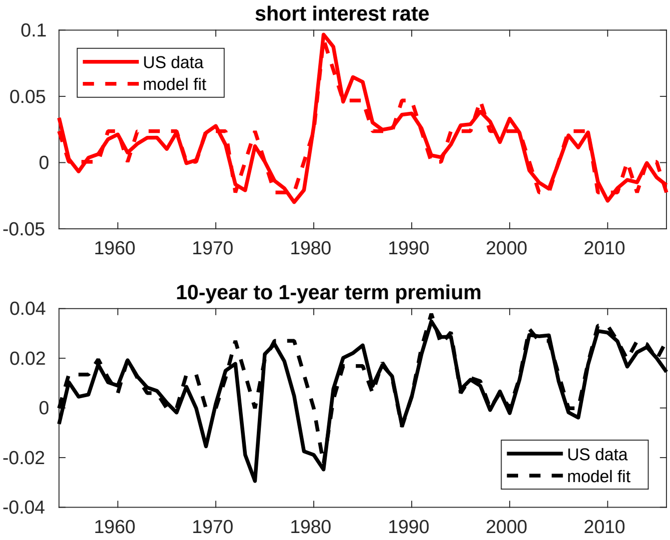
$$r_{t+1} = (1-\rho_r)\bar{r} + \rho_r r_t + \varepsilon_{r,t+1}$$

$$x_{t+1} = (1-\rho_x)\mu_x + \rho_x x_t + \varepsilon_{x,t+1}$$

$$a_t = \alpha_0 + \alpha_1 x_t$$

- Shocks follow a joint normal distrib. $\boldsymbol{\varepsilon_t} \sim \mathcal{N}\left(0, \begin{bmatrix} \sigma_r^2 & \sigma_{rx} \\ \sigma_{rx} & \sigma_x^2 \end{bmatrix}\right)$.

Target joint dynamics of short rate & term premium

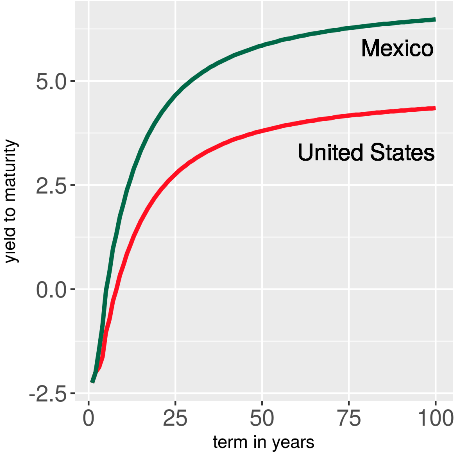


Lenders' stochastic discount factor

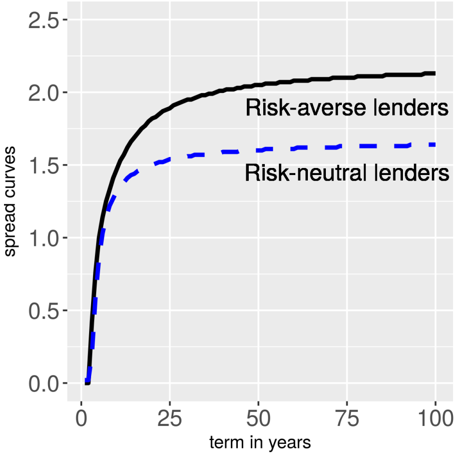
Symbol	Meaning	Value	Source
$\bar{r}$	mean 1-yr rate	0.035	US real 10-year bond yield
ρ_r	persist. 1-yr rate	0.926	
σ_r	st.dev. 1-yr rate	0.015	
α_0	price of risk level factor	-3.11	Joint calibration
α_1	price of risk slope factor	16.59	
μ_x	mean US fundament.	0.21	
ρ_x	persist. US fundament.	0.28	
σ_x	st.dev. US fundament.	0.07	
σ_{rx}	corr. 1-yr rate & US fund.	-0.28	

Calibration targets	Model	Data
mean US 10/1 yr term premium	0.011	0.01
persist. US 10/1 yr term premium	0.44	0.414
st.dev. US 10/1 yr term premium	0.012	0.012
persist. US 1-yr yield	0.79	0.84
st.dev. US 1-yr yield	0.017	0.018
corr. 1-yr yield & 10/1 yr term premium	-0.574	-0.528

Yield and spread curves in the model

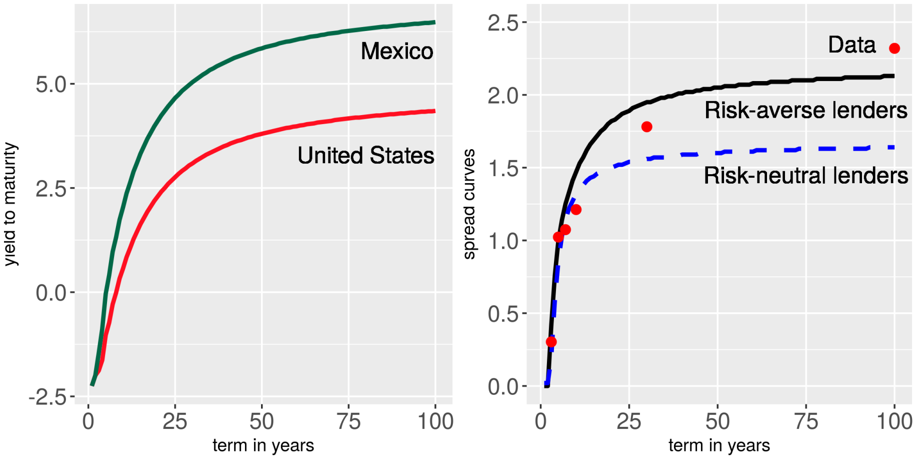


(a) Yield curves



(b) Spread curves

Yield and spread curves in the model



(a) Yield curves (b) Spread curves